6

Cloud-Resolving Models

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A cloud-resolving model (CRM) allows performing numerical simulations of convective clouds such as shallow cumulus, stratocumulus or storms and squall lines with a resolution on the order of a few tens of metres to a few kilometres over a limited-area 4D (time and space) domain. The development of such models over the past decades is briefly reviewed and their specific features are presented. The latter include a non-hydrostatic dynamic and parametrizations of subgrid turbulence, microphysics and radiative processes. The capabilities of such models are discussed based on comparisons with observations and model-intercomparison studies. CRM are used in a variety of ways, from the exploration of cloud phenomenology and process-understanding studies to the development of algorithms for satellite products, as well as to address climate issues and to develop convective and cloud parametrizations for large-scale models. Selected results illustrating this wide utilization are presented. The continuous increase of computer power induces fast changes in modelling perspectives and therefore influences the developments and utilizations of CRM. This is discussed together with emerging scientific questions which will further benefit from CRM simulations.

6. 1. WHAT IS A CLOUD-RESOLVING MODEL?

6.1.1. A model resolving convective moist phenomena of transient nature

Most people have observed more than once in their life that shallow cumulus clouds frequently arise from a clear sky on fair weather days, experienced heavily precipitating storms, or complained about stratocumulus decks dimming sunshine. These are all common meteorological phenomena and they are associated with the development of atmospheric circulations whose space and time scales typically range from a few tens of metres to a few hundreds of kilometres and a few minutes to several hours (these scales are traditionally referred to as micro and mesoscale). These circulations are strongly coupled to moist thermodynamic and microphysical processes (formation and growth of liquid droplets and ice particles, melting of snow, evaporation of rain drops...) and characterized by an asymmetry between narrow and strong in-cloud vertical motions associated with latent heat release and wider, weaker fluctuations taking place in their clear-sky surroundings. As opposed to larger-scale atmospheric circulations, these transient motions are characterized by strong fluctuations of vertical velocity, in other words the hydrostatic equilibrium brakes down at these finer scales where the convective dynamics, i.e. turbulent vertical motions, the moist physics, including condensation of water vapour, formation of precipitation, evaporation..., and their mutual interactions play a major role.

As a result, the modelling of these familiar but highly non-linear phenomena turns out to be particularly challenging in practice. Only limited insight can be gained from analytical approaches because of the very nature of the processes. Precious guidance is inferred from observations but observations alone are generally too limited to provide definite responses to the numerous questions raised by transient convective clouds. Indeed, the first detailed in-cloud aircraft data of the 60's and 70's already revealed a complex reality that was departing in many ways from the hypotheses or concepts underlying the first simplified models of clouds. For instance, the formulation of mass exchanges between cloud and their surroundings was questioned by Warner (1970) – see also Malkus et al. (1953). Several decades later, this topic is still the object of debate and active research (Siebesma 1995, Jonker et al. 2008, Romps 2010, De Roy et al. 2013).

Still, in the late 60's and even later, it seems that fully parametrized models of cumulus clouds were considered by several researchers as a more fruitful avenue than the first attempts to simulate them numerically in a more explicit way (e.g. see the comment of Simpson et Wiggert (1969) on the work of Ogura (1963) and others, also Redelsperger pers. comm.). Against such pessimistic perceptions though, the research carried out in the following 40 years led to the development of several numerical models that explicitly simulate unsteady convective clouds, and these models are now widely utilized in the atmospheric and climate research community as will be seen below.

They are often referred to by the two acronyms LES and CRM, for large-eddy simulation and cloud-resolving model (note that other acronyms such as CEM for cloud ensemble model, or CSRM for cloud system-resolving model, also found in the literature, refer to CRM as well). By design, a LES, or similarly a CRM, is a numerical model whose grid-spacing is fine enough to allow explicit simulations of individual clouds, throughout their whole life cycle or over part of it.

In atmospheric sciences, the distinction between LES and CRM can be viewed as largely historical. It roots in the parallel developments of two types of explicit cloud models dedicated to the studies of smaller and shorter-lived shallow cumulus versus wider and longer-lasting deep convective clouds. With respect to turbulent motions, the theoretical foundations of LES were more clearly defined from the start, as LES were designed to resolve turbulent motions down to the inertial subrange (cf. Chap. Stevens and Siebesma, see also Bryan et al. 2003). However, beyond differences in their formulation of subgrid processes (including turbulent motions, but also microphysical and radiative processes), the underlying equations of CRM and LES are close, and the distinction between the two now often refers to the utilization of finer versus coarser grids in numerical simulations. Indeed, the spatial resolution of a simulation is also intuitively framed by the object under study. It is typically around 1 km in CRM simulations of deep clouds and 100 m in LES simulations of shallow cumulus and stratocumulus. Considerations of the same type dictate the choices of the domain size and duration of a simulation. Thus, an hour of simulation performed over a 10 km-wide and 5-km high domain is typically well suited to model a few coexisting shallow cumulus and to sample their individual life cycles. A 100-km wide 20-km high domain and several hours of simulation become necessary when focussing on deep convective cloud cells reaching the top of the troposphere. Note that these numbers are more indicative of lower space and time limits; nowadays, LES and CRM are often used to perform simulations over much wider domains and longer durations. Note also that in the past, many LES incorporated parametrizations of the microphysics referred to as 'warm' (i.e. which discarded processes involving the solid phase of water), simply because it was not necessary to study shallow cumulus clouds in the Tropics. On the other hand, the importance of ice-phase microphysics to deep convection, which typically extends far above the 0°C isothermal, was identified quite soon, and sophisticated parametrizations of ice-phase processes were introduced in some CRM in the 80's (Lin et al. 1983).

6.1.2. A numerical tool to further process understanding and to explore scientific questions

Figure 6.1a displays a schematic view of a cloud population over Tropical Ocean inferred from observations (Houze and Betts 1981) and Figure 6.1b a three-dimensional snapshot of an ensemble of clouds independently simulated with a CRM. Figure 6.1b was not at all drawn to mimic Figure 6. 1a, still both figures share a number of similarities such as the growing deep convective cells ahead of the system and the thick anvil-type cloud at the rear. A first major interest of the simulation is to provide space and time varying (four-dimensional) fields of both temperature, water vapour and cloud and rain water, horizontal and vertical wind fields together with the detail of the numerous acting processes.

Such a comprehensive set of information on transient convective phenomena cannot be obtained from observations alone. Furthermore, a number of diagnostics can be derived from CRM simulations, via the analysis of budget equations (Figure 6.2) or the use of tracers to follow parcel trajectories, sensitivity tests can be performed too. For instance, one may explore the importance of the humidity field on the structure, strength and vertical extent of convection by comparing simulations using different initial water vapour fields, or test the impact of evaporative processes on the strength of convection by either allowing or suppressing them. Indeed, numerous fundamental questions remain about convective clouds. The factors accounting for their spatial structure, for their spectrum of size and spacing, are not all well understood nor possibly fully identified. Still, well defined recurrent mesoscale geometric patterns are observed, and they are often quitespectacular. They take the form of fair weather scattered cumulus clouds which materialize open cells rooting in the convective boundary layer, or appear as lines of cloud streets or lines of “pearls on a string” (Kuettner 1971). In contrast to scattered cumulus, stratocumulus fields often display closed cell structures (Atkinson and Zhang 1996, see also Wood and Hartmann 2006). Deep convection sometimes aggregate into wider multicellular structures, an archetypical example being the squall-line with its dense line of deep convective cells ahead of a wide and thick stratiform anvil. LES and CRM are powerful tools to address this wide range of fundamental scientific questions within a tightly controlled framework, as illustrated in section 4.

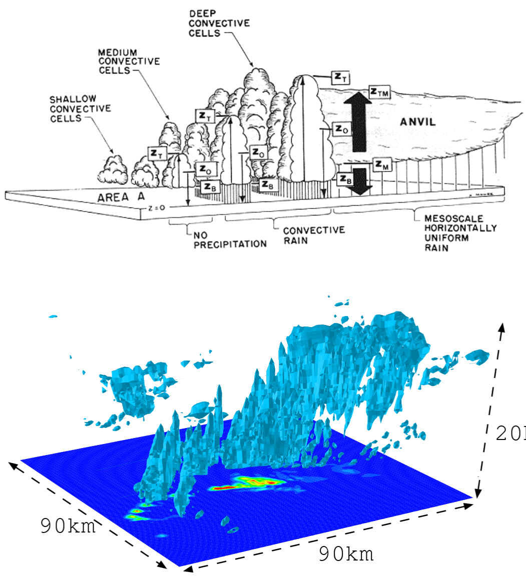

Figure 6.1: (a) Schematic of a typical population of clouds over a tropical ocean. Thin (thick) arrows represent con-vective (stratiform) scale updraughts and down-draughts (from Houze et Betts 1981) while heavy convective and lighter stratiform rain are indicated by respectively narrow and wider hatchings. (b) Three-dimensional view of a cloud field simulated by a CRM, using the isosurface of hydrometeor mixing ratio 0.32g.kg-1, surface précipitation is also shown below with shading (Guichard et al. 1997).

The strength of such simulations is to provide an *explicit* representation of the clouds and associated motions arising at scale larger than the smallest resolved motions (namely a few hundreds of metres at best for a CRM), but one must not mistake these simulations with reality, nor substitute them to observations. Indeed, we know from aircraft in-situ data that turbulent fluctuations are still observed within clouds at scales smaller than 100 m (e.g. Warner 1970). Sub-grid scale motions, together with microphysical, and sometimes radiative processes, are taken into account in CRM, but implicitly, via parametrizations that are presented in section 3. Observations are also of major importance: they help to evaluate and, whenever necessary, improve parametrizations, they also provide major guidance to design and assess the relevance of other important aspects of the simulation, for instance the choice of initial and boundary conditions[1]. Conversely, CRM simulations provide a precious tool to interpret observations which are often sparse and incomplete with regards to the transient nature of convective cloud-related processes and to the questions at hands.

Finally, another growing type of utilization of CRM simulations is dedicated to the improvement or development of new parametrizations of convective processes, in short results from CRM are taken into account and used for guidance. Provided that enough care is taken in the comparison of explicit and parametrized simulations, and importantly, that the focus remains on robust features of the CRM simulations, valuable inferences can be obtained. The CRM is then used as a *numerical laboratory* to further the understanding of interactions among processes and to design more physically-based formulations for parametrized models. The development of single column model versions of large-scale weather or climate models integrating the same set of parametrizations greatly helped in the success of this approach, because the comparison of the results obtained with single-column and cloud-resolving models become a much more direct one (Randall et al. 1996).

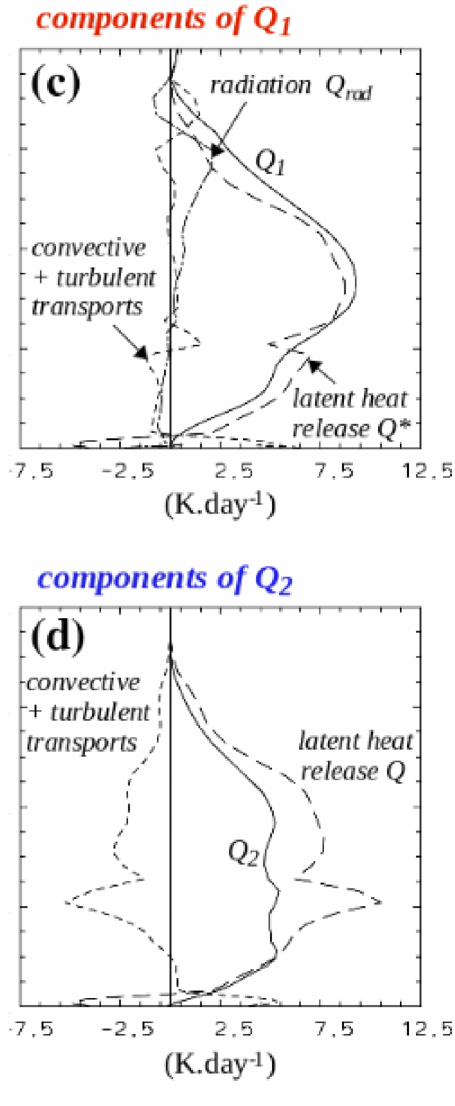

Figure 6.2: An example of the vertical structure of the convective processes operating in temperature and water vapour budgets within an atmosphere experiencing deep convection over tropical ocean.The solid line correspond to the apparent heat source and moisture sink (Q1, Q2) that are parametrized in large-scale models; the dashed, dotted, and dashed-dotted lines indicate respectively the total latent heat release due to microphysical processes, the impact of turbulent and convective transport, and radiative cooling. Differences between Q* and Q are the more pronounced around the height where temperature crosses 0°C due to melting and fusion processes which are affecting the temperature but not the water vapour budget. In this case, above a shallow boundary layer (about 500m thick), the magnitude of radiative and convective processes is relatively small, and the temperature budget is dominated by the latent heat release. However, convective motions, which transport water vapour from the lower to the upper troposphere,

[1] *Note also that LES simulations of convection have often been compared to laboratory analogues; e.g. with water tanks, but there is no straightforward analogues for cumulus convection.*

have a profound impact on the vertical structure of the water vapour budget. They finally account for a substantial part of the difference between the profiles of Q1 and Q2; i.e. the vertical structures of heating and drying in the free troposphere is not solely explained by microphysics. Note also that in the boundary layer, the cooling and moistening induced by the evaporation of precipitation (the only microphysical process operating in this layer) is associated with convective transports (notably those involved in the dynamics of cold pools spreading at the surface). (Results from a CRM simulation, the graphs show 7-day mean profiles over an area 256 km wide, from the surface up to 16 km.)

6.1.3. where CRMs stand with respect to emerging high-resolution modelling systems

Now that CRM simulations of squall-lines can be performed with a LES-type resolution (of about 100 m, Bryan et al. 2003), the distinction between these two types of models tends to become blurred, even if LES of shallow convection are performed with finer and finer resolution (e.g. Matheou et al. 2011).

The situation is even more confusing when considering the evolutions of larger-scale models. Indeed, until very recently, the resolution of numerical weather forecast models (several tens of km) was too coarse for them to explicitly resolve any convective processes and these processes were indeed represented with parametrizations. However, nowadays, several of them employ a horizontal grid size of a few km and, accordingly, they have modified their equations, switching to a non-hydrostatic dynamic of the atmosphere. One can also think of recent simulations performed over domains several tens of degree wide with CRM (e.g. Marsham et al. 2013). Some global simulations have even been performed with such a configuration (Tomita et al. 2005, Satoh et al. 2008). Note also the existence of conceptually more complex types of GCMs; these are embedding a CRM within all of their atmospheric columns (Khairoutdinov et al. 2005), an approach advocated by Randall et al. (2003) and using the so-called super-parametrization framework initially imagined by Grabowski (2001).

The emerging overlap between CRM and GCM indicates that the 'traditional' view of what a CRM stands would benefit from some clarifications. Alternatively, a CRM could be defined as a model within which the fine-scale non-hydrostatic motions and their interactions with physical processes (microphysics, radiation) are explicitly taken into account, regardless of the model being a narrow limited-area model dedicated to mesoscale studies or a global model allowing in-depth studies of the interactions and couplings between convective processes and the global circulation.

The examples above indicate that increase of computation power opens the door to new types of approaches for studying moist convective processes. However, even if this power was to increase to the point that all GCM were able to afford a 1-km grid in the close future, limited-area 'traditional' CRM would certainly continue to be useful in the future for many reasons. First it remains a well-suited tool to study processes and mechanisms within simplified frameworks, for academic purposes and also to interpret more complex models. More fundamentally, numerous issues still need attention and further developments in current CRM. This notably includes (i) the parametrization of boundary-layer turbulent motions (with a 1 km resolution, these motions are partly resolved, partly parametrized, and their representation in CRM is not yet satisfying - e.g. Honnert et al. 2011), (ii) radiative processes (for instance in most CRM, radiative processes are treated independently within each individual column without any possible interactions), and (iii) last but not least microphysical processes. This discussion will be extended later, but we first step back in time to the first pioneer explicit simulations of cumulus clouds.

6.2. BACK TO THE ORIGINS

In meteorology, the acronyms LES and CRM appeared respectively in the 80's and 90's, but the development of these types of models can be traced back to the 60's and 70's. This process is briefly recollected below.

6.2.1. First LES of shallow cumulus clouds and stratocumulus decks

The first Large-Eddy Simulation of trade wind cumulus was achieved by Sommeria (1976), and rested upon extending a model first developed by Deardorff (1972) for the simulation of the dry (i.e. cloud-free) convective boundary layer. In order to simulate cumulus clouds, Sommeria (1976) introduced a parametrization of condensation and evaporation processes together with an additional prognostic equation for a cloud water mixing ratio, with liquid water assumed to take the form of cloud droplets in suspension advected with the flow. With this model, he was able to simulate shallow cumulus clouds with a horizontal resolution of 50m over a 2km-wide domain for five hours (Figure 6.3). Albeit simple, the set up of the simulation was not unrealistic. Initial conditions were cloud-free, with thermodynamic and dynamic profiles derived from a radiosounding, and a given value of SST was prescribed at the surface. The model also took into account longwave radiation and a prescribed larger-scale flow. With these settings, a statistical steady state was reached after an hour of simulation. This pioneer study also showed that heat and moisture fluxes were highly variable in time in the cloud layer (in relation to condensation and evaporation processes) and evidenced the presence of subsidence at the edge of individual clouds. During this same decade, Asai and Nakamura (1978) also developed a two-dimensional LES that provided qualitatively similar results.

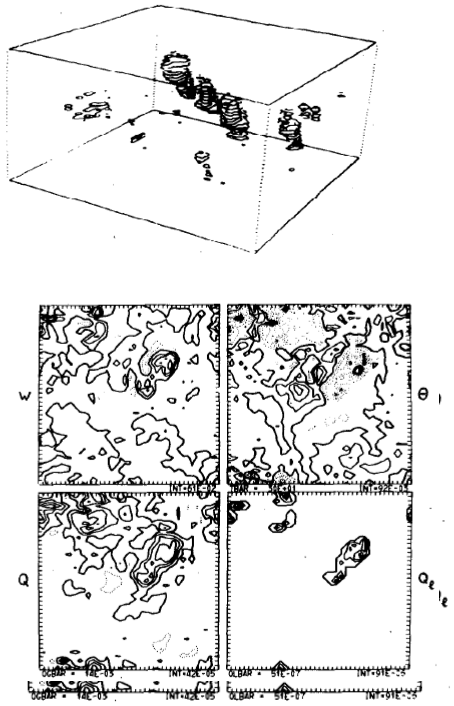

Figure 6.3: illustration of one of the first LES of shallow cumulus clouds (Sommeria 1976); (a) 3D view of the cumulus simulation after 4 hours of simulation. The domain is a 2km-cube and the resolution is 50m in all three directions. (b) horizontal cross sections at 775m of the vertical velocity, the potential temperature, the water vapour mixing ratio and the liquid mixing ratio; these cross-sections highlight the strong correlations occurring among those variables.

Shortly later, a parameterization of subgrid condensation was introduced in the model by Sommeria and Deardorff (1977), assessing that even with a relatively fine mesh on the order of 50m, the assumption that such a mesh should be entirely saturated or entirely unsaturated was crude, a finding further corroborated by observations (Sommeria and Lemone, 1978). This subgrid scheme simply assumed gaussian distributions for the liquid potential temperature and the total water mixing ratio in the mesh, and work is still ongoing nowadays to better depict small-scale distributions of thermodynamic variables in cloud fields. For example, Bougeault (1981) highlighted the interest of a skewed distribution, with a long flat tail related to shallow convection. Joint probability distribution function of vertical velocity, liquid potential temperature and total water vapour mixing ratio (Larson et al, 2002 among others) has also been proposed as a subgrid cloud scheme in CRM. Note however, that the importance of the subgrid cloud scheme increases from LES to CRM.

Improvements in the parameterization of the subgrid turbulence were further carried out by Redeslperger and Sommeria (1981) with the introduction of a prognostic equation of the turbulent kinetic energy and the utilization of thermodynamic variables approximately conserved when water changes phase, and therefore more suited for the formulation of the interactions between turbulent motions and moist thermodynamics.

At about the same time, Deardorff (1980) also simulated a stratocumulus-capped mixed layer over land with this model, albeit with modifications at the lower boundary in order to account for the distinct balance of turbulent fluxes over a land surface. In this study, he explored in particular the role of cloud-top radiative cooling with a suite of sensitivity tests including dry and 'smoke cloud'[2] topped boundary layers as well as stratocumulus decks that were not interacting with radiative processes, and notably concluded that future simulations should use a finer vertical spacing than the 50m used here in order to properly simulate the processes occurring near the inversion and to avoid truncation errors.

Later, Redelsperger and Sommeria (1986) introduced a formulation of precipitation processes (following Kessler 1969), which made use of a new prognostic equation for a rain water mixing ratio variable (the latter departing from cloud water in that rain droplets fall with respect to the fluid they are embedded in). In a similar spirit, Krueger (1988) developed a two-dimensional precipitating cloud model with great care taken to the formulation of turbulence (in this case with the first implementation of a third-order scheme in such models). According to their grid size (~1km), the simulations presented in Redelsperger and Sommeria (1986) and Krueger (1988) can be viewed as of CRM-type. However, both models were either developed from, or inspired by LES of clouds, with particular care taken to the representation of subgrid-scale turbulence, and this tight connection differentiate them from other CRM developed at that time.

In the following decade, various developments improved these models. For instance, Kogan et al. (1995) introduced an explicit formulation of the microphysical processes based on the explicit prediction of a droplet size distribution function (the drops are distributed among different sizes and drops in each size are subjected to advection by wind, condensation, sedimentation...). This approach has a large computational cost which explains why so few LES models used one but was motivated by the need of a better description of the drizzle process. In term of numerics, Raasch and Schroter (2001) presented the first LES run on a massively parallel system with distributed memory. This

[2] *Where a 'smoke cloud' can be thought of as a cloud which is radiatively active, but where no water phase nor microphysical processes change takes place.*

opens the possibility of longer simulation over a larger domain in order to tackle new scientific questions such as mesoscale organization and regime transition. New set-up was also proposed such as a lagrangian approach in order to analyse the transition from the stratocumulus to the trade-wind cumulus regime. Krueger et al (1995) and Wyant et al (1997) proposed to simulate the air motions in a 2D-domain moving with the mean boundary-layer wind and interacting with a changing environment (characterized by varying sea surface temperature, free tropospheric temperature and mixing ratio or mean subsidence).

In the last thirty years, LES have been commonly used using idealised or more realistic set-up. Examples of the main results obtained from those simulations are presented in section 4.

6.2.1. From CRM modelling of convective cells to LES of squall lines

In the 70's, the novelty of the first ancestors of the models now referred to as CRM were lying in their formulation of the non-hydrostatic dynamics. This allowed an explicit treatment of the couplings arising between convective-scale motions, thermodynamic and microphysical processes. As for LES, in practice, this also meant introducing and solving new prognostic equations for vertical velocity and cloud water, but in addition, also for rain water. In retrospect, several aspects of these models, of the simulations carried out at that time, may appear rudimentary. One must keep in mind that the computing capabilities were considerably less than today though. Even more critical, it was necessary to first solve numerous theoretical and numerical difficulties, from the definition and discretization of well suited, tractable equations, including the formulation of appropriate initial and boundary conditions to the introduction of parametrizations for microphysical processes, and again, under the constraint of limited computing power. In fact, by demonstrating the relevance and potential of this numerical approach, these pioneer works paved the way to the subsequent development and further utilization of this type of modelling to study convective clouds.

For instance, Miller and Pearce (1974) developed one of the first three-dimensional non-hydrostatic models of deep precipitating convection. Such a model proved to be able to simulate the development of a single deep convective cell within a 15 kilometres wide domain extending up to the tropopause, from a local perturbation added in the lower levels of an otherwise horizontally homogeneous initial atmospheric state. Note that, by design, this small-size domain precluded the simulation of the interactions arising between deep convection and the larger-scale circulations. Still, the simulation highlighted the strong couplings arising at small spatio-temporal scales between convective motions and microphysical processes, the importance of water loading to the cloud dynamics, and of rainfall evaporation to downdraught formations (these two last points are discussed in more details, from equations, in the next section).

In the following decade, these models benefited from numerous numerical and physical improvements (e.g. Klemp and Wilhelmson 1978), and by the early 80's, they started to be used to study the dynamics of deep convective clouds, the links between their morphology and the wind field, the role of convectively-generated outflows on subsequent convective developments, or the splitting of convective storms and generation of new cells (Wilhelmson et Klemp 1981).

It is worth noticing that several cloud models were built in the 70's and 80's across the world. The distinct underlying objectives leading to their developments readily translated into some differences in their numerical schemes, formulations of initial and boundary conditions, and into various degrees of sophistication in their physical parametrizations. In the 80's, these models were further used to study the morphology and life cycle of individual storms, or to explore the mechanisms at play in wider mature squall lines, in particular the drivers of their self-sustained nature, with numerical simulations lasting a few hours.

The promising capabilities of this new modelling approach also motivated other utilizations which led to additional developments. For instance, some models or model configurations were specifically designed for the purpose of studying the main features and sensitivities of not a single storm but ensembles of deep clouds evolving within, and interacting with, a wider larger-scale environment (Tao and Soong 1986, Krueger 1988, Gregory and Miller 1989, Xu et al. 1992, Held et al. 1993,, Tompkins and Craig 1998). Such a configuration allowed to address the sensitivity of the simulated convective atmosphere to SST, large-scale wind field, microphysical processes with a new, much less parametrized-type of model than those previously used in the past, such as the one-dimensional single-column approach pioneered by Manabe and Wetherald (1967). For these type of studies and others, the need of wide-enough environments and long duration runs led to the design and frequent use of two-dimensional (2D) simulations in the 80's and 90's. This choice may appear surprising because such a framework fails to reproduce the inherently three-dimensional structures of convective processes. Still, it provided an explicit treatment of the tight couplings arising between convective motions, microphysical and radiative processes. Beyond the fact that a 2D CRM presents a number of obvious limitations (Grabowski et al. 1998, Tompkins 2000), its utilization proved to be fruitful to advance on some issues that were finally not much affected by these limitations, or at least less than by other more critical choices. Nevertheless, with the continuously increasing computing power, performing three-dimensional simulations has become more affordable.

6.3. FORMULATION OF THE MODEL

6.3.1. Non-hydrostatic dynamics, prognostic clouds and other chief features

In terms of dynamics, both LES and CRM share an important feature, they are both non-hydrostatic types of models .The *non* in *non-hydrostatic* reminds that historically, the system of equations adopted by meteorological models made use of the hydrostatic hypothesis. This is highlighted by a scale analysis of the vertical velocity equation:

$$\frac{Dw}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

Here we consider a typical horizontal scale of wind speed U=10 m.s^{-1}, with air density $\rho \sim 1$ kg.m^{-3}, $g \sim 10$m.s^{-2}, and a scale height $\mathrm{H} = 10$ km (on the order of the depth of the troposphere) equivalent to P=10^5Pa. At synoptic scale, the horizontal length scale $\mathrm{L_S} = 1000$ km, with associated time scale $\mathrm{T_{synop}}=\mathrm{L_{synop}}/\mathrm{U}=10^5$s (about one day), and a typical vertical velocity scale is $\mathrm{W_S}=10^{-2}$m.s^{-1}. Thus, in SI units:

$$\partial w/\partial t \sim \mathrm{W_S}/\mathrm{T_S} \sim 10^{-7} \quad \text{and} \quad u_i\,\partial w/\partial x_i \sim \mathrm{U.W_S}/\mathrm{L_S} \sim 10^{-7}$$

$$\text{while} \quad (-1/\rho)\,\partial p/\partial z \sim \mathrm{P}/(\mathrm{gH}) \sim g \sim 10$$

So the fluctuations of w, Dw/dt, can be neglected in the equation above, this is the traditional hydrostatic equilibrium.

At smaller scale now, with $\mathrm{L} = 1$ km and a typical vertical velocity scale W=10m.s^{-1} :

$$\partial w/\partial t \sim \mathrm{W/T} \sim 10^{-1} \quad \text{and} \quad u_i\,\partial w/\partial x_i \sim \mathrm{UW/L} \sim 10^{-1}$$

The fluctuations of *w* are still smaller, but cannot be neglected anymore. From physical considerations as well, at these smaller scales (less than 10 km, Yau 1979), fluctuations of vertical velocity, i.e. non-hydrostatism, correspond to a major expression of atmospheric motions. Besides, theoretical considerations imply that the strengthening of vertical motions arising at smaller scales is less than would occur if the dynamics was purely hydrostatic (with a vertical velocity increase that would be proportional to the inverse of the horizontal scale, Weisman et al. 1997). Therefore both LES and CRM incorporate a prognostic equation for the vertical velocity.

Originally, a Large-Eddy Simulation refers to a numerical model based on the resolution of the Navier-Stokes equations which explicitly simulates turbulent motions. The same definition can be retained for LES and CRM of moist and cloudy atmospheric flows, with turbulence including convective motions arising on scales below a few to several tens of kilometres. These equations are spatially and temporally filtered and the smaller-scale turbulent motions are represented via a parametrization. In LES, the filter width generally coincides with the grid size. Note that a LES differs from a Direct Numerical simulation (DNS), as the latter resolves all scales of motion (for the atmosphere, it means a horizontal resolution of a few cm). Accordingly, DNS are also more expensive and currently hardly tractable to simulate atmospheric flows in the same way as currently possible with LES (DNS are nevertheless very informative for specific focused scopes such as the exploration of thin stratocumulus cloud tops and cumulus cloud edges; e.g. Mellado et al. 2009).

In LES and CRM, it is generally assumed that pressure fluctuations balance rapidly and are negligible in comparison to density or temperature fluctuations, this is the anelastic approximation (Ogura and Phillips 1962). Its main interest is to allow filtering high-velocity waves, sound waves in particular; the pressure is then obtained from an elliptic equation. The main advantage of an anelastic formulation is thus to allow for longer time steps as the high speed of sound waves requires the use of very small time steps in finite difference schemes (due to the CFL criterion). In some cases, density fluctuations can further be considered as negligible in the continuity equation; in other words the air is assumed incompressible (or non-divergent). This is the Boussinesq approximation which is often used in LES of shallow flows, for instance in simulations of turbulent motions confined to the first kilometres of the atmosphere above the surface.

There are some limitations to the anelastic approximation of Ogura and Phillips (1962), in particular with respect to the conservation of mass and energy, and a few dedicated studies attempt to improve its original formulation in current models (Durran 1989, Bannon 1996, Arakawa and Konnor 2009). Note however that a few CRM have been developed from the start without this approximation (e.g. Klemp and Whilhemson 1978, Romps 2008), but in that case, specific numerical methods were developed.

More broadly, several numerical choices are made in the design of a LES or a CRM, from the grid type (for instance the vertical grid, in the simplest case, will coincide with a geometric height, but can also be a terrain following coordinate) to the discretization of the set of equations on the retained space-time grid (e.g. flux conservative formulations, pseudo-spectral discretization) and ordering of the different computations required for each simulated process (where distinction between slow versus fast processes help designing rules). The choice and number of prognostic variables also vary from one model to the other, but a minimal set, well suited to simulated non-precipitating shallow cumulus clouds, typically includes prognostic equations for the three components of the wind (u, v, w), for a temperature variable (e. g. either simply the temperature, the potential temperature, or the liquid water potential temperature), and for two water variables (either specific humidity and cloud water qv and qc, or water vapour and cloud mixing ratios rv and rc; in some models, total water is considered instead of water vapour). The number of 'water' variables is directly related to the microphysical scheme as discussed below.

Finally, note that while the spatial grid in LES are often close to isotropy (dx = dy ~ dz ~ 50-100m), it is generally stretched on the vertical in CRM, with a much finer discretization in the lower levels (less than 100 m) than above (a few hundred m) and dx = dy ~ 1km. Thus CRM grids are often highly anisotropic.

6.3.2. parametrizations of subgrid-scale motions

LES were initially designed to provide a statistical view of turbulent motions and it was assumed that the subgrid-scale turbulence is mainly isotropic and confined to the inertial range whereas the larger eddies are resolved and depend on the environment. Most of the time, subgrid-scale turbulent motions are taken into account by parametrizations based on local arguments with turbulent fluxes in the three directions expressed as a function of local gradients, i.e. :

$$u_i'\alpha'=-K\, u_i\, (\partial\alpha/\partial x_i)$$

where *K* is a so-called eddy-diffusivity coefficient. However, the assumptions underlying such formulations typically break down close to the surface and in stable layers of the atmosphere where turbulent eddies become smaller and much less isotropic. There are mainly two schemes that are used to parametrize subgrid-scale turbulent motions. The first one, initially developed by Lilly (1962) and Smagorinsky (1963) assumes that the buoyancy and shear productions of turbulent kinetic energy balance the molecular dissipation. This leads to eddy diffusivities being proportional to local velocity and temperature gradients and function of a Richardson number (note that the constant of proportionality varies significantly from one model to the other) or expressed as a function of a Richardson number. The second, more sophisticated scheme (Lilly 1967 and Deardorff 1980), introduces the turbulent kinetic energy in the formulation of eddy diffusivities, and requires the resolution of a prognostic equation for the turbulent kinetic energy. Both schemes involve a length scale which is often identified with the grid size except in stable layers where it is reduced to avoid excessive turbulent mixing (e.g. Deardorff 1980).

There are some identified weaknesses associated with turbulence schemes, and more broadly subgrid-scale processes in LES, notably in the simulation of cloud edges, an example being stratocumulus cloud tops, whose dynamics critically involves fragile combinations of parametrized processes (turbulent and microphysical), grid size and numerical filtering (Stevens et al. 1999, 2005). These motivate some recent studies which explore these cloud interfaces with DNS (Mellado et al. 2009, Abma et al. 2013).

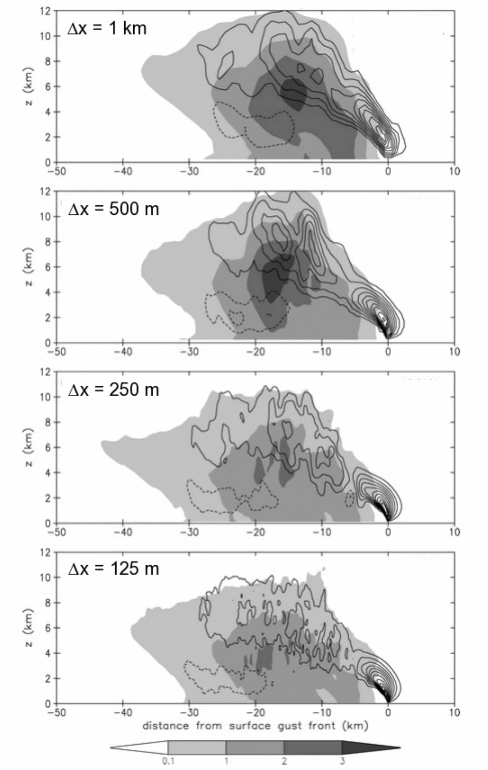

Figure 6.4: Mean vertical cross sections of vertical velocity (contour interval 1 m s^{-1}) and rain water mixing ratio (shaded) of a squall line simulations using grid spacings of (a) 1000 m, (b) 500 m, (c) 250 m, and (d) 125 m in all three directions (except for (a) where the grid spacing is 500m on the vertical). (adapted from Bryan et al.2003).

In the past decades, and still now, numerous CRM simulations have been performed with Smagorinsky-type schemes (e.g. Tompkins and Craig 1998, Romps 2008) that were initially designed for finer-grid LES. Klemp and Wilhelmson (1978) were already quite aware of the weakness of this formulation for CRM as the coarser grid-size is then lying outside of the inertial range. One can guess these authors and others thought that it was premature to deal with this problem at that time, and that other difficulties had to be solved first. The study of Bryan et al. (2003) illustrates the sensitivity of the simulation of a squall line to the grid size (Figure 6.4) : as the mesh is refined, from 1 km to a LES-type resolution of 125m, the vertical velocity weakens somehow, in particular in the stratiform part (to the left); and rainfall decreases (grey shading). However, it is obvious too that the main features of the squall line are already present in the 1 km simulation. Furthermore, it is important to realize that such simulations are typically as much influenced by the formulation of the microphysics (a simple warm Kessler-type scheme in this study) and its interaction with subgrid-scale motions. This issue of representation of subgrid motions in CRM is also documented by Takemi and Rotunno (2003) who further emphasize how purely numerical filtering was dominating over subgrid-scale turbulence in their model when using standard values of the parameters involved in the turbulent scheme.

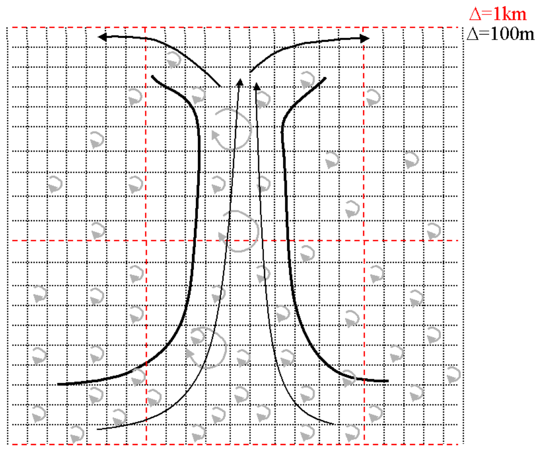

Figure 6.5: Schematic of a generic boundary layer thermal (black lines and arrows) and small-scale turbulence (grey arrows) with typical grid meshes of LES (CRM) indicated by the black (red) dotted lines (adapted from Cuxart et al. 2000 & Hourdin et al. 2002).

However, even with less numerical filtering and more advanced turbulent schemes (e.g.; a TKE equation is more often considered now), several issues remain. This concerns the simulation of subgrid-scale moist dynamics (for instance at cloud edges), but not only. In particular, boundary-layer convective thermals are reasonably well resolved by LES but marginally so with the coarser CRM grids (Figure 6.5), this is a so-called *grey zone* where both resolved and parametrized motions are active, with issues associated with 'work-sharing'. For instance, the atmosphere is very reactive to 'dry' convective instabilities, and if the subgrid mixing is not strong enough to remove it quickly, spurious thermals can develop instead at the larger resolved scales (Honnert et al. 2011), which can further lead to unrealistic structures in simulated boundary-layer cumulus clouds (dictated by these resolved boundary-layer motions). In fact, for CRM as well as coarser-grid mesoscale models, local eddy-diffusivity formulations are not well designed to deal with convective boundary layer turbulence, because it manifests as a non-local process (thermals scale with the boundary-layer depth, irrespective of the -smaller- grid size) with counter-gradient turbulent fluxes. Local eddy-diffusivity formulations, by design, tend to underestimate such fluxes and can not reproduce counter-gradient fluxes. In order to solve this issue, non-local schemes (e.g. Troen and Mahrt 1986, Hong and Pang 1996) are sometimes implemented (e.g. Wu et al. 1998), and modification of the mixing length have also been proposed (Bougeault and Lacarrère 1989). More recently, boundary layer mass-flux schemes (e.g., Hourdin et al. 2002, Neggers et al. 2009), which provide a more explicit and mechanistic representation of this turbulent process, have been implemented in some CRM (e.g.; Pergaud et al. 2009).

6.3.3. Water phase changes and microphysics

Note that simulations of shallow non-precipitating clouds do not all incorporate an explicit representation of microphysical processes. Instead, condensation and evaporation of water are dictated by a moist thermodynamic adjustment. Thus, cloud water forms when the air is saturated, with the underlying assumption that the concentration of cloud condensation nuclei is large enough so as not to delay condensation. However, microphysical considerations become necessary as soon as one focusses on rain formation, or mixed phase clouds for instance.

The more frequent formulation of microphysical processes in LES and CRM is based on an a-priori separation of hydrometeors into two main categories: (i) cloud water, including small liquid droplets and ice cristals suspended within the air mass, and (ii) precipitating water, either in the form of rain drops, hail, graupels, aggregates or snow, each falling with respect to the air mass. This distinction between different types of hydrometeors is somewhat artificial, arbitrary, especially for ice-phase hydrometeors, and other approaches are explored to move away from these hypotheses (Morrison and Grabowski 2008). However, as they stand now, current parametrizations of microphysical processes typically include prognostic equations for each retained hydrometeor-type, from a single one (for simulation of non-precipitating clouds) to two (for liquid phase precipitating clouds) to four or more (for deep convective clouds).

Microphysical processes are numerous and complex, and this translates into exchanges between the different types of hydrometeors which are controlled by several tens of parametrized processes when the solid phase is considered, among which condensation, autoconversion, accretion, evaporation, melting, riming, ice initiation and deposition, snow aggregation, sedimentation (e.g.; Lin et al. 1983, see Figure 6. 6).

A bulk approach is often adopted in current LES and CRM, which means that particle-size distributions are specified (as opposed to a bin approach). A classical example, still in use today, was proposed by Marshall and Palmer (1948) for warm rain, and is expressed as $n(D) = n_0\, e^{-\lambda D}$, where n is the density of particles, D the diameter of the particle, λ is referred to as the slope parameter and n_0 as the intercept parameter[3]. Likewise, the

[3] *Note that a more general distribution function,*

parametrization of rain formation proposed by Kessler (1969), which expresses in a simple way the formation of raindrops by autoconversion and accretion of cloud droplets, is also a basis for numerous CRM in use today.

Microphysical parametrizations are traditionally referred to as single-moment when they incorporate prognostic equations for the mixing ratios of the different hydrometeor types, and as double-moment when they also integrate prognostic equations for particle number concentrations. By design, the latter is more flexible, and better suited for studies focussing on drizzle formation and aerosol-cloud interactions. LES of shallow warm clouds now often make use of two-moment schemes (e.g.; Khairoutdinov and Kogan 2000, Seifert and Beheng, 2006). Attempts to introduce these more complex schemes in CRM simulations of deep convection exist (Meyers et al. 1997, Milbrandt and Yao 2005, Phillips et al. 2007), they notably point to a sensitivity of rainfall evaporation and convective cold pools to these one- versus two-moment formulations (Morrison et al. 2009).

More broadly, behind these important distinct features among models, there still exist a wide diversity in the content of microphysical schemes in LES and CRM, from the specification of hydrometeor types (e.g. particle size distributions, mass-diameter relationships), the types of processes taken and not taken into account, their formulation... Overall, this field is still the object of active research as developed in Chapter X. Numerous studies performed in the past decade show that LES provide a valuable platform to test and implement new parametrizations of microphysical processes (Larson et al, 2002). Furthermore, they allow exploring the sensitivity of clouds to these processes within a dynamic framework where they actually interact with small-scale turbulent motions (Stevens and Seifert 2008). On the other hand, when using a LES or a CRM, the choice of an appropriate microphysical scheme may appear delicate; the goal of the study can frame to some extend the level of sophistication of the scheme though. For instance, when focussing on convective cold pools and precipitation, it is worth keeping in mind that these features are sensitive to the size and fall speed of rain droplets, as noted above. Likewise, processes driving the formation and dissipation of cloud ice are also important for cloud radiative effects.

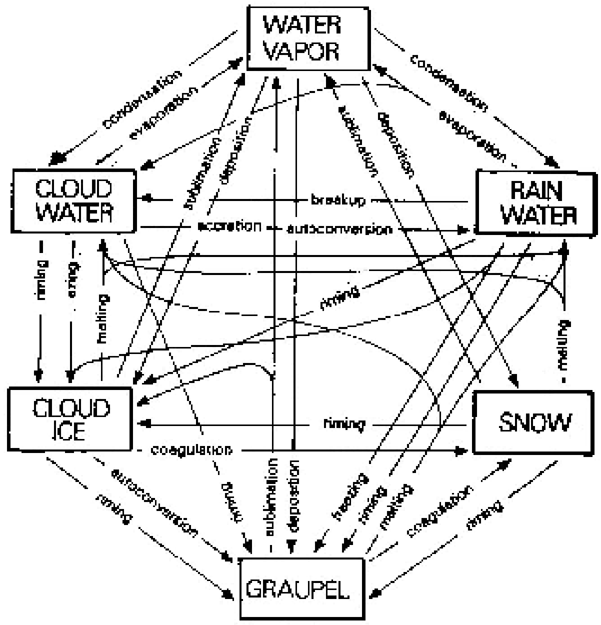

Figure 6.6: An example of schematic summarizing for a given microphysical scheme, the selected water categories or species, with arrows indicating the different processes operating among these species (for instance condensation which tranform water vapour to cloud (liquid) water. (source: R. Forbes)

Cutting the atmosphere into grid meshes introduces artificial discontinuities in the operation of cloud processes. In the simplest case, these are simply ignored and each grid box is either unsaturated or saturated, there is no consideration of subgrid-scale microphysical processes nor of any partial cloud cover that could interact with radiation. This "all or nothing" hypothesis can become quite unrealistic, for instance the resolution of a CRM does not allow an explicit representation of shallow cumulus clouds. In that case, clouds may well develop, but typically later and too big from the start, as dictated by the numerics. The existence of thresholds in microphysical parametrization (e.g. Kessler rainfall scheme) also introduces a sensitivity of the results to the resolution. In order to limit such numerically-driven sensitivities, parametrizations have been developed which account for subgrid-scale processes, i.e. turbulence and microphysics, with a partial cloud cover that can be inferred from the latter. 2nd-order scheme are formulated in terms of subgrid-scale variances and covariances of thermodynamic and dynamic fields. In the past case, gaussian distributions have often been assumed (e.g. Sommeria and Deardorff 1977, Mellor 1977), and several studies are still working now at improving the realism of these distributions (Golaz et al. 2002, Bogenschutz et al. 2013).

expressed as $n(D) = n_0 D^{\alpha} e^{-\lambda D}$, *a Gamma function, is however often considered for solid hyrometeors, following Ulbricht (1983)*.

6.3.4. Radiative processes

It may appear surprising that numerous LES and CRM cloud simulations have been, and still are, performed without much consideration of radiative processes. In fact, this appears as a reasonable assumption in some cases, for instance for the simulation of short-term internal dynamics of cumulonimbus clouds, because for such phenomena, the radiative heating rates are typically of much smaller magnitude than convective processes. However, as soon as one aims to perform simulations over longer time scales (typically more than a few days), or focus on some types of convective clouds (e.g., stratocumulus, or ice anvils), the neglect of radiative processes and of their interactions with convective motions and microphysics can become dubious.

The formulation of radiative processes in LES and CRM spans very diverse flavours and ranges of accuracy, in part as a result of the important amount of computing time required by the parametrization of this process, but also for methodological purposes. For instance, let's consider a simulation of daytime boundary layer clouds over land using prescribed surface sensible and latent heat fluxes at the lower boundary. It is frequent practice to neglect atmospheric radiative processes in such simulations (e.g. Neggers et al. 2003)[4]. However, even if the *direct* interactions of radiative processes with atmospheric motions and clouds are not taken into account, still, their (major) imprint on the surface-driven boundary layer growth is expressed in surface turbulent heat fluxes, as the surface energy balance dictates that the sum of these fluxes equate net radiation minus the ground heat flux (H+LE=Rnet-G).

The presence of transient clouds largely affect radiative cooling rate in both the shortwave and longwave at small time and space scales, but one can recall that a daily-mean value is typically 1-2 K.day^{-1} in the Tropics. Thus, radiative processes actively contribute to the atmospheric heat balance at scales of a few days and more, as temperature does not fluctuates much. This radiative constraint is sometimes introduced in a very simple way (e.g. Robe and Emanuel 1996, Muller et al. 2011). In both studies, CRM simulations are carried out over tens of days to mimic convective equilibrium states over ocean using fixed SST and constant radiative cooling rates in the absence of any larger-scale advection. Basically, such a setup ensures that convective activity does not cease and helps to prevent temperature and moisture drifts that could otherwise occur. Note however that cooling rates as large as 5 K.day^{-1} as sometimes found in the literature cannot be interpreted as a formulation of radiative processes, because such a rate appears much stronger than indicated by physically-based considerations of radiative processes. More fundamentally, one must realize that by design, such simplification excludes the operation of any cloud-mediated convective-radiative interactions.

Even when interactions between cloud and radiative processes are critical to the cloud dynamics such as for stratocumulus, simplified or empirical formulations are sometimes employed to relate radiative heating rates to liquid water mixing ratios. For instance, Stevens et al. (2005) made such a choice because it was better suited for an LES intercomparison whose object was not to explore the sensitivity of the results to the parametrizations of radiative processes as such.

In the past 20 years, more sophisticated parametrizations of radiative processes have been progressively introduced in several LES and CRM though in order to account for the cloud reflection, absorption and scattering of radiation. They often resemble parametrizations used in large-scale models, with separate formulations of longwave (LW) and shortwave (SW) radiation (with major control of scattering in the SW while emission and absorption dominate in the LW). As in large-scale models, these radiative transfer schemes also consider a limited number of spectral bands (referred to as broadband schemes). They usually make use of the two-stream approximation whereby the radiative flux divergence are expressed in each band as a difference between an upwelling and a downwelling radiative flux, and these fluxes are computed independently for each model column (with no horizontal exchanges). Note that the situation is conceptually simpler in CRM and LES than in large-scale models because the cloud field is now almost fully resolved; the overlap assumptions, which are needed to decide how to arrange the different cloud layers within a column, are therefore only relevant to the remaining columns that contain grid cells where the cloud cover is less than unity (and only when a subgrid cloud cover is parametrized).

Unlike in large-scale models where precipitating hydrometeors are often removed from the atmosphere by parametrized convection as soon as they form, precipitating hydrometeors are explicitly simulated in LES and CRM and span some time into the atmosphere before they reach the surface or experience evaporation for instance, so that they can, in principle, participate to radiative transfer. However, their impact is often neglected and only the radiative properties of cloud liquid drops and ice crystals (plus sometimes drizzle and snow) are taken into account.

The impact of clouds on radiation is framed by the size distribution, shape and concentration of the hydrometeors they are made of. Thus, the radiative properties of liquid -close to spherical- drops are typically better ascertained than those of ice particles, which display wide variations in shapes and sizes, even within a single cloud.

Practically, in the shortwave, radiation schemes include a formulation of the optical thickness (σ), single scattering

[4]*The same applies to simulations of the dry convective boundary layer.*

albedo (ω) [5] and asymmetry factor (g) [6] in cloudy pixels for each separate band. These three variables are most often expressed as a function of the liquid or ice cloud water path (CWP) and of an effective radius (r_e)[7], following some parametrizations used in large-scale models (e.g. Slingo 1989, Ebert and Curry 1992, Fu 1996). For instance, in Slingo (1989) and Ebert and Curry (1992), they are expressed as:

$$\boldsymbol{\sigma}_i = CWP\,(a_i + b_i/r_e)$$
$$1\text{-}\boldsymbol{\omega}_i = c_i + d_i\, r_e$$
$$\boldsymbol{g}_i = e_i + f_i\, r_e$$

where the subscript i refers to the i^{th} spectral band and a_i, b_i, c_i, d_i, e_i and f_i are fitted parameters. Note that simpler parametrizations have been proposed too, that only retain a dependence on the cloud water path (e.g. Sun and Shine 1995).

For the longwave, in the simplest case, scattering is neglected and clouds are treated as grey body with an emissivity parametrized as $\varepsilon = 1 - e^{\alpha\, LWP}$, where α is a constant on the order of 0.15 (Stephens 1978). Typically a thick convective cloud almost behaves as a black body with $\varepsilon \sim 1$, unlike a thin cirrus cloud. Note that more refined parametrizations have been develop that also account for the not-always-negligible impact of cloud scattering in the longwave (e.g. Fu et al. 1998).

A basic issue associated with the utilization of such parametrizations in models is that they were developed for specific types of clouds, sometimes designed with the help of a few experimental data, and they may not be as appropriate when applied to other cloud types. In addition, for those parametrizations where the effective radius r_e stands as an input variable, some ad-hoc choices are sometimes made (e.g. in the simplest case a fixed value of r_e), without direct connection with the parametrization of cloud microphysics. Even if someone is not working specifically on these different schemes and on their interactions, it can turned out to be important to know about these 'details' in order to correctly interpret simulations and their sensitivities.

Finally, one must also realize that the underlying assumptions of two-stream parametrizations become less and less valid as the geometries of clouds depart more from shits of plane parallel layers (e.g. cumulus clouds). In fact, the few studies addressing how such simplified formulations compare to fully 3-dimensional radiative transfer computations in cloudy skies tend to support the usefulness of two-stream parametrizations (see Pincus 2013). Still, the integration of computationally-expensive radiative processes in high-resolution LES of clouds raises numerous new challenges (e.g.; Pincus and Stevens 2009).

6.3.5. Initial and boundary conditions and their significance for limited-area modelling

Intuitively, one conceives the importance of the initial conditions to the modelling of atmospheric cumulus clouds. For instance, the vertical development of deep convective cells are constrained by the atmospheric stability and tropopause height, while their morphology is strongly shaped by the wind field, notably the vertical shear. However, the choice of boundary conditions is often as crucial, even for short-duration (less than a day) runs, and it must not be overlooked when designing LES of CRM simulations. So far, we discussed the need to properly represent subgrid-scale turbulent motions (together with the physical processes and their couplings). However, LES and CRM results are also quite sensitive to larger scale motions as well, and the latter are set or specified by the choice of lateral boundary conditions.

For part of it, this sensitivity to initial and boundary conditions reflects a framing of large-scale states and circulations on convective processes, and to quote a common expression, is sometimes interpreted as "the *response* of convection to larger-scale processes". Still, this sensitivity can also express the strong interactions arising between convection and the larger-scale environment; they can be especially fast, in terms of vertical motions, when deep convection is operating.

As a kind of rule, people designing and using LES and CRM have often tried to formulate initial and boundary conditions that are well suited to the purpose of the simulations while remaining as simple as possible. Hereafter, we recall commonly used initial and boundary conditions, but these are not exclusive at all, and other choices can be imagined as long as they are meaningful and suited to the purpose of a simulation.

A simple and frequent choice is to initialize the simulation with atmospheric profiles applied homogeneously to each vertical column of the domain, a domain whose horizontal size is typically less than the scales of fluctuations in the free troposphere (at least for calm, non-convective conditions). To do so, and depending on the purpose of the simulation, profiles of temperature, moisture and horizontal wind can be derived from a sounding, or designed from more academical considerations. The initialization of vertical velocity and cloud water contents is more delicate (a simple choice is to

[5] *The single scattering albedo ω is the ratio of scattering to extinction (sum of scattering and absorption); ω ranges from 1 for purely scattering particles to 0 when extinction is solely due to absorption. For clouds, it is typically well above 0.5.*

[6] *The asymmetry factor is an indicator of the direction of scattering with $g = -1$ ($g = 1$) for backward (forward) scattering.*

[7] *The effective radius r_e is the ratio of the third to the second moment of the particle size distribution. In the case if (non-spherical) ice particles, r_e is estimated using spheres of equal surface area.*

set them to 0).

From these horizontally-homogeneous initial profiles, small-scale motions are often initiated by some noise applied in the low levels. However, the simulation of convection with this approach can be unsuccessful when the sources of convective instability (e.g. surface boundary conditions and more specifically surface fluxes, radiative processes, vertical motions) are not well taken into account. This is typically the case for the simulations of deep convection that ignore surface heat fluxes, and it may happen that deep convection never occurs or only after several hours of simulations, in an environment that has drifted too much from the initial state. This is the main reason why in some simulations of deep convection, warm of cold 'bubbles' are 'added' to the initial homogeneous vertical structure. These are typically less than one km deep and one to a few km wide and aim to mimic in an academic way the triggering of convective cells by a thermal or a gravity current (Klemp and Whilhemson 1978, Bryan 2003, Yeo and Romps, 2013). Once initiated, for instance with a line of cold bubbles, these deep cells can feed in turn a cold pool and thus, further sustain deep convection. Such a simulation setup is well suited to study a deep convective event in its mature stage; while by design, it precludes investigations of the mechanisms of convective initiation (as these are enforced).

Finally, note that the real atmosphere is turbulent, not horizontally homogeneous. Therefore, when a simulation is initiated with such an approach (with either random noise or isolated bubbles added), a certain amount of time is necessary for realistic turbulent, convective motions to develop; this is the spin-up period. Its length depends on the time scale of the processes under study; for instance it is reasonable to discard the first hour of simulation when focussing on convective boundary-layer processes, which corresponds typically to a few turnover times.

Then, both LES and CRM are limited-area domain, so that lateral boundary conditions are required. The two most common choices are periodic and open lateral boundary conditions. The former is well suited to numerically explore the behaviour of a convective phenomenon of relatively large spatial extend, but which displays regular patterns. For instance, when focussing on wide decks of stratocumulus or shallow cumulus fields extending over hundreds of km, it can be both convenient and meaningful to choose to simulate a small piece of it, then viewed as a part of a wider homogeneous system, with periodic lateral boundary conditions (note that the horizontal domain of simulation must be large enough though to contain the cloudy patterns of interest). In the case of periodic boundary conditions along the x-direction for instance, each variable a must satisfy the following, involving the two x-axis borders, for each point y along the y-axis, and at all vertical levels z and time steps t: $a\ (x_{N+1}, y, z, t) = a\ (x_1, y, z, t)$, where N is the number of points along the x-axis, and x_{N+1} is a fictitious point introduced here for numerical purpose.

A direct consequence of this major constrain is induced by mass conservation: it involves that the net horizontal divergence of fluxes across the domain and the mean vertical velocity are both zero at all height. When this assumption corresponds to an unrealistic hypothesis with detrimental effects, a formulation of larger-scale horizontal and vertical advection is often introduced as in Sommeria (1976) or Grabowski et al. (1996). The latter in particular provides a comprehensive presentation of the separation between larger (prescribed) and smaller (simulated and parametrized) scales of motions underlying such derivations. In short, additional terms are introduced into the budget equations of temperature and water variables, corresponding to these larger-scale advection, in the form $-W_{LS}\ (\partial\ \alpha_{LS} / \partial z) - [U\ (\partial\alpha/\partial x)+V(\partial\alpha/\partial x)]_{LS}$. These profiles are then applied homogeneously to all columns of the simulated domain. For instance, a large-scale subsidence is often introduced in LES of subtropical shallow cumulus clouds, and combined with the horizontal-mean profile of simulated α, to formulate the effect of the large-scale vertical advection associated with this subsidence. This additional term can also be used to prevent unrealistic thermodynamic drifts as can arise when the sources of energy to the system (surface fluxes and radiation) are not balancing each other (in terms of equivalent potential temperature, or equivalently, moist static energy). It is also used in simulations of deep convection carried out over large domains where larger-scale advection correspond either to an academic setting or is inferred from observations or meteorological analyses. In this periodic configuration, the large-scale control on the wind field is often taken into account via a nudging of the horizontal-mean wind to given wind profiles (or, sometimes the introduction of a given geostrophic wind, when the Coriolis force is taken into account, as it cannot be represented in the absence of large-scale horizontal pressure gradient). Implicit to this nudging choice is the assumption that the mean wind is largely governed by the larger scale dynamics, and must therefore be prescribed. This is especially important if one wants to reproduce the spatial patterns of convective clouds (for instance scattered convection versus squall-lines, anvils...) which are strongly framed by the mean wind field. This can also be critical to surface energy exchanges, especially over ocean.

Note that periodic conditions are not the best suited to the simulation of all convective phenomena. For instance mature squall lines which develop as isolated mesoscale phenomena displaying cloud shields up to several hundreds of km wide are generally better apprehended by models using 'open boundary conditions'; the aim there is to build somewhat 'transparent' boundaries. In that case, the larger scales of motion are not prescribed, but instead tend to passively 'respond' to convection arising within the domain, and typically an inflow of air develops in the lower troposphere and an outflow above. Note that some care is needed though in the formulation of this type of open conditions in order to minimize wave reflection at the lateral boundaries (e.g.; Klemp and Wilhelmson 1978, Carpenter 1982). A major difference with periodic boundary conditions arises in the budget equations, because the horizontal mean vertical velocity, largely controlled by convective processes developing inside the domain, departs from 0 and accounts for substantial vertical advection. Such simulations are generally carried out over a few to several hours to study specific deep convective systems.

Finally, lower and upper boundaries are also needed and they are generally kept simple in CRM and LES. Both of them often consist in rigid lids (w=0). The upper bound is typically set a few to several km above the atmospheric layer under study, and the introduction of a sponge layer allows to damp wave reflection at the top of the domain. The choice of the lower boundary is generally a more important issue, because it frames the formulation of surface fluxes, which in turn strongly controls the magnitude of boundary layer turbulence. Over ocean, a simple choice is to prescribe values of SST together with bulk formula for surface fluxes. To explore ocean-atmosphere couplings arising at the mesoscale, it is necessary to couple an ocean mixed layer (or even wave and spray models) to the LES, but this area has not received much attention yet. Likewise, over land, it is common practice to assume that the surface is flat, and to prescribe surface sensible and latent heat flux, together with a roughness length. Coupling between the land surface-cloud couplings just start to be addressed with LES (Lohou et al. 2014), and this again requires the coupling of a land surface model; note that it can be kept relatively simple though (e.g. the Penman Monteith model is a good example of a simple land surface scheme). Note that clouds have a strong impact on the surface energy budget, especially on the incoming shortwave radiation, and such a coupled modelling framework may prove to be very helpful in the next decade.

6.4. SELECTED RESULTS

6.4.1. Performing simulations and evaluating them

It is not possible to summarize here the range of simulations performed with CRM and LES, from purely academic to strongly observationally-constrained. Indeed, they are used for numerous purposes, to help addressing a very wide range of scientific questions. Still, evaluating simulations is always necessary, especially as questions become more demanding for models. In this respect, dedicated observational campaigns provide the opportunity to test numerical simulations against the real world, and these have been widely used, from the seventies until now. They often focus on statistical properties of cloudy convective boundary layers, on the scales (in time and space) and magnitude of convective features, rather than on a particular simulated cloud. Examples include comparison between observations and LES of fractal dimension of cloud boundaries (Siebesma and Jonker, 2000) or cloud field distributions (Neggers et al, 2003). Such studies are important but not straightforward, and a true delicacy is related to the design of the evaluation approach, as uncertainties in the initial and boundary conditions can cause departures from observations in the simulations, which are not related to the physics or numerics of the model. This also reflects the inherent difficulties of observing transient intermittent cloud processes. Conversely, note that when simulations are sufficiently close to observations, they provide a precious extension to data alone. A popular and complementary evaluation of LES and CRM started in the eighties as several of these models were developed and used across the world: it consists in model intercomparison, where simulations are performed with different models (with distinct numerics and physical parametrizations) using close setups (in terms of domain size, resolution, time of simulation..., and wherever possible initial and boundary conditions). Such intercomparison are the only way to assess whether different models provide similar (or not) depictions of major variables that are often at best partly obtained from observations (for instance turbulent and convective fluxes, cloud statistics), but critical for the development of process-based parametrizations in large-scale models. Several intercomparison of this type have been carried out, for both shallow and deep convective clouds, over land and over ocean.

For LES of shallow cumulus clouds, beyond a good agreement on the mean variables and flux profiles, some issues remain about the turbulent kinetic energy and representation of cloud tops and precipitation fluxes (Stevens et al. 2001, Brown et al. 2002; Siebesma et al. 2003, Van Zanten et al. 2011), while the simulation of stratocumulus (e.g. Stevens et al, 2005) and deep convective clouds appears more challenging. Figure 6.7 further illustrates one of these intercomparisons for the simulation of a mature squall line (Redelsperger et al. 2000). The four CRM simulate close spatial structures of the convective system, with a convective line at the front (on the left) and a more stratiform region behind, and the overall structure is consistent with observations (upper panel). Note however the lower magnitude of rain water in the fourth CRM (lower right), a departure that at least partly related to the formulation of the lateral boundary conditions (it was the only simulation using periodic instead of open conditions, and the zero horizontal-mean vertical velocity appeared to dump deep convection in this simulation). Note also that the scale of the fine-scale convective structures can be sensitive to choices in the numerics.

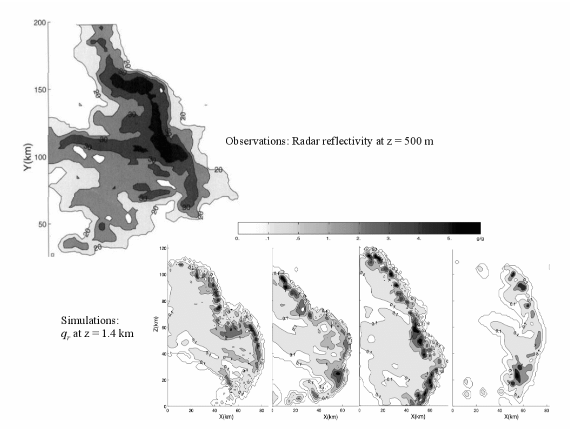

Figure 6.7 Illustration of an intercomparison of CRM simulations of a mature squall line performed with four distinct CRM (lower panels showing specific rain water at 1.4 km above the surface, the upper figure shows the structure of the observed convective system with radar reflectivity 500 m above the surface, for a qualitative evaluation of the simulated spatial structures (adapted from Redelsperger et al. 2000)

Another example is provided in Figure 6.8 from Xu et al. (2002). In that case, the simulation were more energetically constrained, with periodic boundary conditions and prescribed surface heat and latent heat fluxes (i.e. a configuration generally used for LES intercomparison). It illustrates the magnitude of the mean thermodynamic biases that can be expected from this type of simulations (the similar vertical structures of the biases across models suggests again an influence of the -common- boundary conditions). On the other hand, the left panel indicates that all models provide rather close profiles for the convective updraught and downdraught (not available from observations), and emphasizes the significance of the convective downward mass flux, an expected feature in this case portraying deep convective events over land. There is relatively more scatter in the downdraught profiles, and this is again expected as the dynamics of downdraught dynamics, strongly controlled by rain evaporation, relies more heavily on the parametrization of microphysics than the dynamics of updraughts (cf XX) which is controlled at first order by the thermodynamics of phase changes.

Overall, the numerous model intercomparisons have pointed to much more agreement among the depiction of convective cloud processes in LES and CRM than obtained with parametrized models, in terms of mean structures as well as timing (e.g. phase in the diurnal cycle). As a result, the robust 'non-observable' outputs of the simulations turn out to be very helpful in the development of more physically-based parametrizations, and these are now widely used for this purpose.

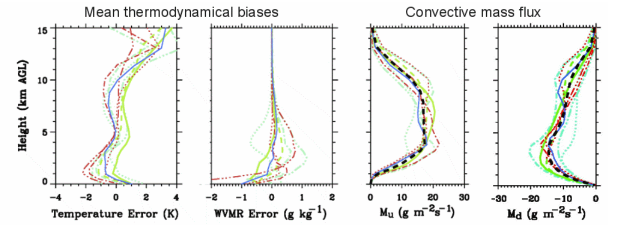

Figure 6.8 Intercomparaison of CRM simulations of 14-day mean profiles of thermodynamic biases relative to observations (left panels) and the convective updraught and downdraft defined in the same way in each run (right panels) - each line corresponds to one run and the thick dotted lines in the right panels are the average of CRM profiles (adapted from Xu et al. 2002)

6.4.2. insights into convective clouds phenomenology and process understanding

As discussed above, the causes behind numerous observed patterns of clouds (open and close cells, structuring -or aggregation- of deep convection in mesoscale multi-cellular deep convective systems, their orientation...) are not all very well understood. In the last decades, several studies have tried to reproduce these structures with LES and CRM, for instance those observed with satellite data in the stratocumulus regions of the south-east border of the oceans which are illustrated below.

In general, cloud systems driven by cooling at their top, such as stratocumulus, organised preferably as closed cells. This horizontal organisation is similar to a Rayleigh-Benard type of organisation with however a ratio of cell diameter to cell height around 20:1 instead of 3:1. Latent heat release associated with condensation, drizzle formation (Xue et al 2008, Savic-Jovic and Stevens 2008) and large-scale dynamics have been invoked to explain this difference. However, the representation of precipitation in stratocumulus is challenging and the size of the observed pattern is demanding in terms of computing power. Recently, the role of atmospheric aerosols has also been highlighted as controlling the formation of precipitation and therefore the mesoscale organisation (Wang et al, 2009). Feingold et al. (2010) showed that the concentration of atmospheric aerosols could control the stratocumulus horizontal organisation from closed cells with a large aerosol concentration associated to weakly precipitating clouds to open cells with a small aerosol concentration associated to precipitating clouds (Figure 6. 9). In the open cells, strong updraughts are present on the cell walls. In those thick clouds, precipitation form and fall. When falling, evaporation of the precipitation induces a cooling and the formation of downdraught movement that creates subsidence in area where a short while ago strong convergence was present leading to the formation of open cells in other place and creating an oscillating system.

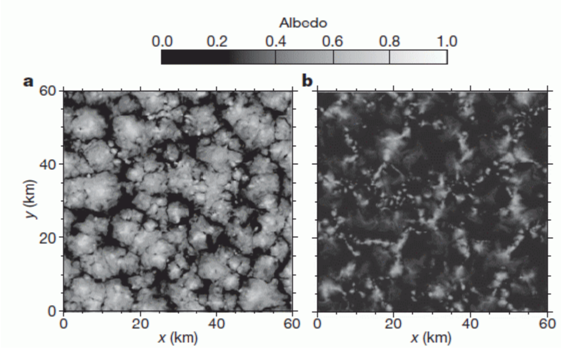

Figure 6.9 Simulated cloud albedo from a LES of (a) closed and (b) open cellular structures. The two simulations only differ in the aerosol concentration at initiation : (a) high concentration favouring non-precipitating clouds and (b) low concentration favouring drizzle (from Feingold et al 2010).

Driven by this same aim to better understand cloud structures and processes, more and more, passive or Lagrangian tracers are introduced in LES and CRM to track the circulation of air in relation to the clouds and to better depict the associated transport. As an example, Zhao and Austin (2005) analysed with passive tracers the life cycle of a few cumulus clouds, each a few hundreds of m deep and lasting less than half an hour, in a simulation of trade-wind cumulus. A first tracer was introduced in the subcloud layer in order to define the area affected by the transport from the cloud that they showed to correspond to roughly 2 to 3 times the area identified by liquid water content. A second one was introduced in an upper layer (as shown in black in Figure 6.10) to analyse the mixing dynamics of each cloud. In particular, they showed that the mixing mainly occurred through turbulent structures present at the ascending cloud top characterized by a complex vortical circulation with a strong ascending branch in the centre of the clouds, a large divergence at the top and subsidence at the edges, consistently with laboratory results of ascending thermals. Heus et al. (2008) also used Lagrangian tracers to explore exchanges of air in-between the clouds and their environment and focused on narrow subsiding shells occurring along the clouds. They identified the evaporative cooling as the main forcing of these structures and suggested their role in compensating the upward mass flux inside the cumulus.

Finally, using passive tracers proved to be an insightful approach for the development of parametrizations (Couvreux et al. 2010, Rio et al. 2010).

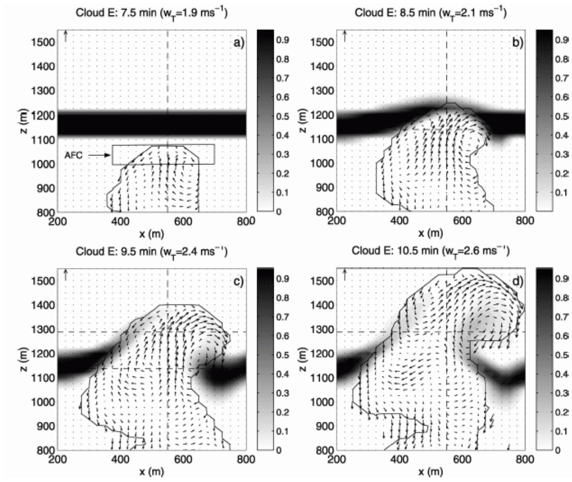

Figure 6.10 Vertical cross-sections centred on the upper part of a growing cloud separated by one min: arrows indicate wind in the cross-section. The contour delineates the cloud determined by area with qc> 0.01g/kg. The shading indicates the mixing ratio of a tracer introduced at t=7.5 min (t=0 coincide with the beginning of the 'cloud life')between 1100 and 1200 m uniformly (1g.kg-1). Starting at t=8.5 min, the cloud penetrates, deforms this layer, transporting tracer upward but mainly on the edge of the cloud (from Zhao and Austin 2005).

Jumping now to deep convective phenomena, LES are also well suited to address the increasing focus on their non-stationary transition phases (e.g. within the diurnal cycle over land) and more broadly, on the mechanisms accounting for the life cycle of transient deep convective systems, notably those arising at mesoscale. For instance, even though the interactions between the convective boundary layer and deep convection have been emphasized for a very long time, it is only nowadays that this issue can be addressed numerically with a resolution that is fine-enough to explicitly simulate boundary-layer thermals and a domain size that is large-enough to contain a convective system.

Observations and such simulations both point to the importance of convectively-generated cold pools which are illustrated in Figure 6.11. It emphasizes the coupled fluctuations of temperature, vertical velocity and wind speed characterizing this phenomenon. In particular, as the cold pools spread into the boundary layer, they generate narrow updraughts at the front of the cold pools, and thus provide an efficient mechanical lifting for initiating new deep cells. Note also the very strong enhancement of wind speed, which, over ocean, can substantially increase surface heat and momentum fluxes, and over arid land, accounts for large uplift of mineral dust. The basic physics behind the dynamics of such cold pools is well understood. However, their interactions with the boundary-layer dynamics (e.g.

how strong BL thermals affect their sharp boundaries and strength) and with surface processes, the precise mechanisms through which they help sustaining further convection (e.g. mechanical lifting versus modification of the thermodynamics), are still poorly known. These questions just start to be addressed with LES now able to provide realistic depiction of such transient sequences, and their representation in large scale models is in its infancy (to date, a parametrization of cold pools has been implemented in one single large-scale model, Hourdin et al. 2013).

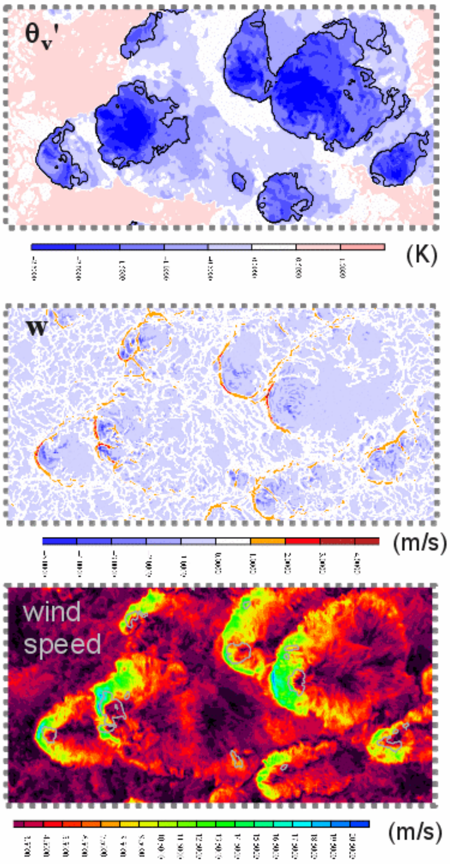

Figure 6.11 Horizontal cross section of potential temperature anomaly (top panel), vertical velocity (middle panel) and wind speed (bottom panel) close to the surface, from an LES of deep convection. The figures illustrate the properties of the convectively-generated cold pools.

6.4.3. Exploring basic climatic issues

Beyond the process-type studies highlighted in 6.4.2, LES and CRM are also extensively used to explore climatic issues, such as climatic feedbacks associated with convection and clouds. Here, these fine scale-models are not considered as a substitute to full GCM. Rather, they are used for complementary insights into these questions. For instance, one can study the impact of idealized climate change perturbations such as a change in SST or an increase in atmospheric CO2 on the cloud cover (does it increase or decrease? and via which mechanism?), on the hydrological cycle (does it rain more or less? does the intensity of rainfall change? and if so, at which scale?). A major interest of this approach is also the possibility to explore the sensitivities of the results to the couplings between physical processes in a comprehensive way.

An archetypal example of an academic concept (or frame) which helps addressing basic climatic issues is the convective-radiative equilibrium (CRE). The CRE connects surface temperature to radiative forcing at large scale with very simplified models of the earth system (Manabe and Wetherald 1967). The CRE frame has been since revisited many times. In its simple form, it consists in a single-column atmospheric model which incorporates a formulation of convective and radiative processes (plus some assumptions at the surface; e.g. a prescribed albedo), and simulations are run until reaching a thermodynamic equilibrium.

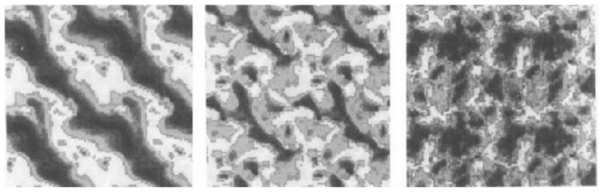

Figure 6.12 An illustration of the sensitivity of the spatial structure (or organization) of deep tropical convection to the coupling versus non-coupling of physical processes in CRE simulations. The plots show the water vapour mixing ratio at the lowest model level for (left) a reference simulation, (middle) a simulation where surface fluxes do not respond to mesoscale fluctuations of the surface wind and (right) a simulation where radiative processes are prescribed instead of computed from the thermodynamical profiles and cloud field. The domain is replicated four times so that each panel represents a 200 km x 200 km square. From Tompkins and Craig (1998).

This type of simulation can be carried out with CRM instead of a single column model, and this was indeed first experienced in the nineties. Typically, these simulations use periodic boundary conditions together with a prescribed SST; the time to reach a thermodynamic equilibrium, driven by radiative processes, is on the order of ten days. One must keep in mind that, even though the interactions among processes are represented in a more explicit way than in fully parametrized models, the content of physical parametrizations has more time to imprint the results than in shorter duration runs. For instance, over ocean, radiative processes can only weekly affect the mean temperature profile in one-day runs, but their contribution becomes much more important when focussing on multi-day timescales. Indeed, Tao et al. (1999) found that differences in the formulation of surface heat flux was largely accounting for the very contrasted temperature and water vapour at equilibrium obtained with two CRM. With these limitations in mind, this framework can still be quite insightful. Figure 12 (left panel) shows the spatial structure of convection in such a CRE simulation where the mean wind and wind shear were weak. However, convection displays mesoscale banded structures which turns out to emerge from interactions between surface fluxes, atmospheric radiation and convective processes (compare with Figure 6. 12, middle and right panel). In particular, taking into account convective-radiative interactions leads to more convergence in the cloudy areas and longer lasting clouds. Note that sensitivities of convective spatial patterns to the wind field can be even more spectacular (Figure 6. 13), and accompanied by changes in the cloud cover, water and energy budgets. There exists no systematic comparison of CRM simulations of CRE. However, studies carried out so far agree on the influence of delicate mechanisms of interactions between convective, clouds and radiative processes as important drivers of the simulated patterns, and of the climatic sensitivities of these simulations.

More recently, another type of lateral boundary conditions, distinct from the CRE, has been introduced in CRM to investigate climatic issues. It is referred to as the weak temperature gradient (WTG, Sobel and Bretherton 2000), and was motivated by the uniformity of temperature profiles observed in the oceanic Tropics, close to the Equator. In short, it operates on lateral boundary conditions, and consists in prescribing a temperature rather than a mean vertical velocity profile for larger-scale advection. Thus, in CRM with periodic lateral boundary conditions, this translates into a mean temperature nudged towards a given profile, which in turns dictates the fluctuations of vertical advection (Raymond et al. 2005). While very different by design from the open lateral boundary conditions presented in 3., this WTG framework also allows more freedom to the operation of convective processes. A questioning result arising from these studies is the existence of multiple equilibrium for a given SST, with a final equilibrium, either dry versus wet, largely controlled by the initial water vapour field (Sessions et al. 2010). Whether this functioning is connected or not to the observed bi-modality of the atmospheric water in the tropics (Zhang et al. 2003) is an open issue which needs further elaboration, but it already renews our current, more static view of tropical convective-radiative equilibrium.

More broadly, the links between the climate sensitivity of the real world and such academic simulations is not always straightforward, but to the least, they allow identifying some of the physically-based mechanisms at play, contribute to advance our general understanding, and provide synthetic climatic fingerprints of simulations.

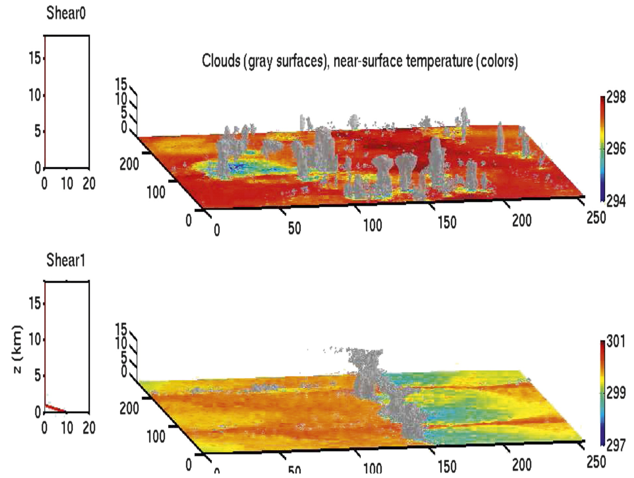

Figure 6.13 Snapshots of clouds and near-surface temperatures in two CRE simulations, one without wind shear (top panel) and the other with some shear (lower panel) – the shear profiles are indicated on the left. The presence of shear changes the spatial scale of convective patterns, with isolated cells replaced by squall-line type systems. (From Muller 2013).

5. FUTURE OF EXPLICIT CLOUD-RESOLVING MODELLING

As extensively discussed above, LES and CRM are fine-scale limited-area numerical models whose major specificity is to provide explicit simulations of the mesoscale dynamics associated with convective clouds. They integrate parametrizations in order to represent major subgrid processes (turbulence, microphysics, radiative processes). However, unlike GCMs, their grid size allow the numerous couplings arising between convective motions and physical processes to be resolved. It took several decades to develop these models to the point where they stand now, encompassing numerous steps of evaluation, refinements and improvements. In the mean time, their utilization proved very fruitful to the understanding of several cloud-related issues that cannot be satisfactorily addressed with observations alone; they are also now widely used as 'numerical laboratory' which guide and help the development of cloud and convection parametrizations for larger-scale models.

It would be misleading though to consider these models as frozen black boxes. First, whenever using such a model, it is necessary to be aware of its formulation, of the thermodynamics and boundary conditions notably, of its parametrizations of physical processes and of their couplings... Second, a large amount of work is still dedicated to their improvement, of their parametrizations and numerics in particular. Additional model developments are also necessary to explore a range of uncovered scientific questions; e.g. couplings with land-surface and ocean mixed-layer models, introduction of dust surface uplift, aerosol, chemistry...

Still, it seems very likely that these modelling tools will be very useful in the next future to address the numerous issues related to surface-atmosphere feedbacks over land arising at different scales (e.g. daytime convective initiation and its sensitivity to surface fluxes and mesoscale heterogeneities), to identify new modes of interactions between mesoscale and larger-scale circulations involving convection and clouds (e.g. synoptic-scale waves and intraseasonal modes of variability), and to pinpoint smaller-scale turbulence-microphysics couplings.

Finally, several perspectives arise from the increasing computing power (even if not the clue to all issues). It allows to perform simulations with either enhanced resolution and/or larger domain sizes and times of integration. Recent progress has already been achieved on this latter front with convection-permitting simulations (grid-size of 4 km x 4 km) carried out over wide regions for several weeks such as performed in the CASCADE project. For instance, Marsham et al. (2013) shows how the monsoon circulation is radically changed over West Africa (and improved in several ways) when deep convection is explicitly simulated. This result involves changes in the diurnal phasing of convection (typically better reproduced in CRM than parametrized models) and a contribution of the vertical mass flux associated with convectively-generated cold pools to the atmospheric low-level cooling, i.e. a process which is distinct from the cooling otherwise operated at larger scale by the monsoon flow via horizontal advection.

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